Solving Rational Expressions (5.5)

- One term Rational Expressions on both sides
  - Cross-multiply and solve using algebra
    - Remember to always check for extraneous solutions (solutions that would make denominator equal to zero)
  - Ex> $\frac{9}{3x} = \frac{-6}{10}$ $x \neq \cancel{0}$ $x = 5$
  - Ex> $\frac{x-3}{5} = \frac{8}{x}$ $x \neq \cancel{0}$ $x = 8, -5$

CROSS MULTIPLY

1) $9 \cdot 10 = -6 \cdot 3x$

$$\frac{90}{-18} = \frac{-18x}{-18}$$

2) $5 \cdot 8 = x(x-3)$

$40 = x^2 - 3x$

$-40 \qquad\qquad -40$

$0 = x^2 - 3x - 40$

FACTOR $(x-8)(x+5)$

Solving Rational Expressions (5.5)

- Equation with more than one term on each side
  - Remember to always check for extraneous solutions (solutions that would make denominator equal to zero)
  - 2 step process
    - 1) Mult. each piece of the equation by the LCM (numerator only). This will eliminate all denominators
    - 2) Solve using Algebra
- Ex> $\frac{10}{3} = \frac{4}{x} + 2$ $x \neq \cancel{0}$ $x = 3$

LCM is $3x$

$3x\left(\frac{10}{3}\right) = \left(\frac{4}{x} + 2\right) 3x$ THIS ELIMINATES DENOMINATORS

$10x = 12 + 6x$

$-6x \qquad\qquad -6x$

$$\frac{4x}{4} = \frac{12}{4}$$

Solving Rational Expressions (5.5)

- Extraneous Solutions
  - Once you solve for the value of x, ensure the answer DOES NOT RESULT IN 0 IN DENOMINATOR
  - Ex> $\frac{2x-5}{x-8} + \frac{x}{2} = \frac{11}{x-8}$ $x \neq 8$ $x = -4$
  - Ex> $\frac{16}{x^2-16} = \frac{2}{x-4}$ $x \neq \pm 4$ No Solution $(x+4)(x-4)$

1) LCM $2(x-8)$

$2(x-8)\left(\frac{2x-5}{x-8} + \frac{x}{2}\right) = \frac{11}{(x-8)}(2(x-8))$

$2(2x-5) + (x-8)x = 11 \cdot 2$

$4x - 10 + x^2 - 8x = 22$

$x^2 - 4x - 32 = 0$

$(x-8)(x+4) \qquad x = \cancel{8}, -4$

BUT $x \neq 8$

2) LCM: $(x+4)(x-4)$

$(x+4)(x-4)\left(\frac{16}{x^2-16}\right) = \left(\frac{2}{x-4}\right)(x+4)(x-4)$

$16 = 2x + 8$

$-8 \qquad\quad -8$

$\frac{8}{2} = \frac{2x}{2}$ $x = 4$ BUT $x \neq 4$

1

Review
And/Or Statements

- When solving rational inequalities you must remember the your answers will be a range of answers. These answer are written as and/or statements. Just like quadratic inequalities.
- AND   OR

$$\# \leq x \leq \# \qquad\qquad x \leq \# \quad OR \quad x \geq \#$$

- Be Careful because the extraneous solutions must not be an equals to inequality.

ONLY BETWEEN WHEN
AND
IF IT IS OR YOU NEED
2 INEQUALITIES

AND: IN THE MIDDLE / IN BETWEEN
OR: OUTSIDE

Rational Inequalities (5.5)
Range of Answers

- Solving Using Graph and Table (Calculator)
  - Plug left side into Y_1
  - Plug right side into Y_2
  - Look at graph and table for where condition is met
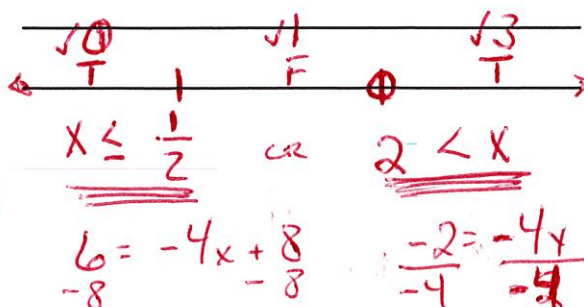 - Ex> $\dfrac{x}{x-4} \leq 2$ Ex> $\dfrac{x}{x-5} \leq 0$

WHEN LOOKING AT CALCULATOR
→ TABLE: X=4 = ERROR
  → BECAUSE CAN'T DIVIDE BY 0
  → THIS IS 1 LIMIT - look for OTHER BY COMPARING VALUES
→ OTHER END IS X=8 ⇒
  VALUES ARE X<4 or X≥8

2ⁿᵈ EQUATION: 0 ≤ X < 5

Rational Inequalities (5.5)
Range of Answers

USE NUMBER LINE

- Solving using Algebra (5 steps)
  - 1)Determine excluded values (denominator = 0)
  - 2)Set the inequality into an equation
  - 3)Solve the equation
  - 4)Put all numbers on a number line (excluded & solutions)
  - 5)Check regions for when condition is met
- Ex> $\dfrac{6}{x-2} \geq -4$

· 2 is EXCLUDED

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① DETERMINE EXCLUDED VALUE (LIMIT)
② FIND LCD
③ MULT BOTH SIDES BY LCD
④ SOLVE WITH ALGEBRA
⑤ CHECK FOR TRUE/FALSE

$$X \leq \frac{1}{2} \quad \text{or} \quad 2 < X$$

$$\frac{6}{x-2} \geq -4$$

$$\begin{array}{cc} 6 = -4x + 8 & -2 = -4y \\ -8 \quad -8 & -4 \quad -4 \end{array}$$

2

Practice B
Solving Rational Equations and Inequalities

Solve each equation.

1. $x - \dfrac{6}{x} = 5$

2. $\dfrac{15}{4} = \dfrac{6}{x} + 3$

3. $x = \dfrac{3}{x} + 2$

4. $\dfrac{4}{x^2 - 4} = \dfrac{1}{x - 2}$

Solve each inequality by using a graphing calculator and a table.

5. $\dfrac{6}{x + 1} < -3$

6. $\dfrac{x}{x - 2} \geq 0$

7. $\dfrac{2x}{x + 5} \leq 0$

8. $\dfrac{-x}{x - 3} \geq 0$

Solve each inequality algebraically.

9. $\dfrac{12}{x + 4} \leq 4$

10. $\dfrac{7}{x + 3} < -5$

11. $\dfrac{x}{x - 2} > 9$

12. $\dfrac{2x}{x - 5} \geq 3$

Solve.

13. The time required to deliver and install a computer at a customer's location is $t = 4 + \dfrac{d}{r}$, where t is time in hours, d is the distance, in miles, from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 6.2 hours for the employee to deliver and install a computer for a customer located 100 miles from the warehouse, what is the average speed of the delivery truck?

Holt McDougal Algebra 2

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Practice B

Solving Rational Equations and Inequalities

Solve each equation.

1. $x - \dfrac{6}{x} = 5$

LCD: X
X ≠ ∅

X = -1, 6

2. $\dfrac{15}{4} = \dfrac{6}{x} + 3$

LCD = 4X
X ≠ ∅

X = 8

3. $x = \dfrac{3}{x} + 2$

LCD: X
X ≠ ∅

4. $\dfrac{4}{x^2 - 4} = \dfrac{1}{x - 2}$

CROSS MULTIPLY
X ≠ 2
SOLVE TO X = 2 BUT ∅ SO NO SOLUTION

Solve each inequality by using a graphing calculator and a table.

5. $\dfrac{6}{x + 1} < -3$

-3 < X < -1

6. $\dfrac{x}{x - 2} \geq 0$

X ≤ 0 OR X > 2

7. $\dfrac{2x}{x + 5} \leq 0$

-5 < X ≤ ∅

8. $\dfrac{-x}{x - 3} \geq 0$

∅ ≤ X < 3

Solve each inequality algebraically.

9. $\dfrac{12}{x + 4} \leq 4$

10. $\dfrac{7}{x + 3} < -5$

11. $\dfrac{x}{x - 2} > 9$

12. $\dfrac{2x}{x - 5} \geq 3$

Solve.

13. The time required to deliver and install a computer at a customer's location is $t = 4 + \dfrac{d}{r}$, where t is time in hours, d is the distance, in miles, from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 6.2 hours for the employee to deliver and install a computer for a customer located 100 miles from the warehouse, what is the average speed of the delivery truck?

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Name: Period: Date:

Practice Worksheet: Solving Rational Equations

Solve each equation and check for extraneous solutions. You must show work and your answers must be correct to get credit.

Level 1	Level 2	Level 3
1] $\dfrac{x}{4} = \dfrac{9}{4x}$	5] $2 = \dfrac{x+2}{x-3}$	9] $\dfrac{x^2+3}{7x} = \dfrac{x+1}{6}$
2] $\dfrac{x}{4} = \dfrac{x+2}{2}$	6] $\dfrac{x}{2x+1} = \dfrac{2x}{x+2}$	10] $\dfrac{2}{x^2-x} = \dfrac{1}{x-1}$
3] $\dfrac{4}{x} + 1 = \dfrac{2x+2}{x}$	7] $\dfrac{9}{x} - 1 = \dfrac{3}{x} + 2$	11] $\dfrac{x^2}{3x-1} + 2 = \dfrac{2(x-3)}{3x-1}$
4] $\dfrac{2x}{x-2} + \dfrac{1}{x+2} = \dfrac{10}{x^2-4}$	8] $\dfrac{x}{x-1} - \dfrac{1}{x-2} = \dfrac{2x-5}{x^2-3x+2}$	12] $\dfrac{x}{2x-1} - \dfrac{2}{2x+1} = \dfrac{x^2+20}{4x^2-1}$

Name: _____   Period: _____   Date: _____

Practice Worksheet: Solving Rational Equations

Solve each equation and check for extraneous solutions. You must show work and your answers must be correct to get credit.

Level 1	Level 2	Level 3
1] $\frac{x}{4} = \frac{9}{4x}$ $x \neq 0$ $\frac{4x^2}{4} = \frac{36}{4}$ $x = \pm 3$	5] $2 = \frac{x+2}{x-3}$ $x \neq 3$ $2x - 6 = x + 2$ $x = 8$	9] $\frac{x^2+3}{7x} = \frac{x+1}{6}$
2] $\frac{x}{4} = \frac{x+2}{2}$ $2x = 4x + 8$ $x = -4$	6] $\frac{x}{2x+1} = \frac{2x}{x+2}$ $x \neq -\frac{1}{2}, -2$ $x^2 + 2x = 4x^2 + 2x$ $-x^2 - 2x \quad -x^2 - 2x$ $0 = 3x^2$ $x = 0$	10] $\frac{2}{x^2-x} = \frac{1}{x-1}$
3] $\frac{4}{x} + 1 = \frac{2x+2}{x}$ LCD: X $x \neq 0$ $4 + x = 2x + 2$ $x = 2$	7] $\frac{9}{x} - 1 = \frac{3}{x} + 2$ LCD: X $x \neq 0$ $9 - x = 3 + 2x$ $-3 + x \quad -3 + x$ $6 = 3x$ $x = 2$	11] $\frac{x^2}{3x-1} + 2 = \frac{2(x-3)}{3x-1}$
4] $\frac{2x}{x-2} + \frac{1}{x+2} = \frac{10}{x^2-4}$ $2x^2 + 4x + x - 2 = 10$ $x \neq \pm 2$ LCD $2x^2 + 5x - 12 = 0$ $(x+2)(x-2)$ $(2x-3)(x+4)$ $x = \frac{3}{2}, -4$	8] $\frac{x}{x-1} - \frac{1}{x-2} = \frac{2x-5}{x^2-3x+2}$ $x \neq 1, 2$ LCD: $(x-1)(x-2)$ $x^2 - 2x - (x-1) = 2x - 5$ $x^2 - 5x + 6$ $x = 3, \neq$ $(x-3)(x-2)$ only 4	12] $\frac{x}{2x-1} - \frac{2}{2x+1} = \frac{x^2+20}{4x^2-1}$

11. $2 < x < \dfrac{9}{4}$ 12. $5 < x \le 15$

13. About 45.5 miles per hour

Practice C

1. $r = -\dfrac{4}{9}$ 2. no solution.

3. $x = 7$ and $x = -1$ 4. $d = \dfrac{1}{5}$

5. $x < -1$ or $x > 0$ 6. $-5 < x \le -3$

7. $-3 < x \le -2$ 8. $x < 3$ OR $x > 4$

9. $m < 0$ or $m \ge 4$ 10. $5 < s < 9$

11. $z \le -24$ or $z > 4$ 12. $x < -12$ or $x > 15$

13. About 14.83 in.   14. About 18.6 h

Reteach

1. $x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x = -4,\ x = 2$

2. $x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3,\ x = -2$

3. $x(x) = 4(x) + \dfrac{5}{x}(x)$

$x^2 = 4x + 5$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$x = 5,\ x = -1$

4. $\dfrac{x + 1}{x + 2} \cdot 5(x + 2)$

$= \dfrac{x}{5} \cdot 5(x + 2)$

$5 + 5(x + 1) = x(x + 2)$

$x^2 - 3x - 10 = 0;\ x = 5$

5. $\dfrac{x}{3} \cdot 3(x - 1) + \dfrac{x + 3}{x - 1} \cdot 3(x - 1)$

$= \dfrac{4}{x - 1} \cdot 3(x - 1)$

$x(x - 1) + 3(x + 3) = 12$

$x^2 + 2x - 3 = 0;\ x = -3$

Challenge

1. $x < -5$ or $x > -3$ 2. $x < \dfrac{1}{2}$ or $x \ge \dfrac{7}{5}$

3. $-3 < x < 2$

4. $x < -3$ or $-1 < x < 1$

5. $x < -1$ or $1 \le x \le 2$ or $x > 3$

6. $x < -1$ 7. $-6 \le x < 5$

8. $\dfrac{1}{8} < x < 5$ 9. $x < 2$ OR $x > 7$

Problem Solving

1. a. $\dfrac{1}{j}$

b. $\left[\dfrac{1}{4}(2.5)\right] + \left[\dfrac{1}{j}(2.5)\right] = 1$

c. $6\dfrac{2}{3}$ h

2. a. $\dfrac{1}{n}$

b. $\left[\dfrac{1}{n}\left(\dfrac{1}{3}\right)\right] + \left[\dfrac{1}{\frac{1}{2}}\left(\dfrac{1}{3}\right)\right] = 1$

c. 1 h

3. C   4. A

Reading Strategies

1. $x = 2$ 2. $x = 0$

3. $x = 3$ 4. $x = -1, 0, 1$

5. $x = -5, 0$ 6. $x = -\dfrac{2}{3}$

7. $x = -3, 3$ 8. $x = 0, \dfrac{1}{4}$

9. $x = -2, 1$

10. No; possible answer: the solution should be $x \ge 1$ or $x < \dfrac{1}{2}$, $x = \dfrac{1}{2}$ is an extraneous solution.